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ON STOCHASTIC TIME-COST-QUALITY TRADE-OFF IN PROJECT SCHEDULING

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Abstract

In this paper, the quality parameter is embedded in the traditional time-cost trade-off problem under some environmental uncertainty in order to develop a time-cost-quality trade-off problem (TCQTP) in a multi-phase project. To do this, we propose a new approach based on goal programming and robust optimization formulation to deal with the problem. To the best of our knowledge, this problem has not been extensively treated in the literature yet. Computational results which are presented in the following sections show the applicability and usefulness of the method.

Key words: *Time cost, quality trade-off, Stochastic optimization, Uncertainty*

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Introduction

Time, cost and quality are usually three interdependent crucial objectives which are often randomly traded off in project practices in the absence of effective tools.

In a project scheduling, it is by and large taken to shorten duration of some activities through consuming extra budget in order to achieve shorter project completion time. This approach that can be taken under either some pre-defined budget or a threshold of project completion time, is known as time-cost trade-off problem (TCTP). This problem leads to a counterbalance between the project completion time and its' total cost. According to (Salmasnia et al., 2012), some TCTP studies can be classified regarding to some modelling factors such as: 1) stochastic or deterministic environment that forces the network to be CPM (Hazır et al., 2010a), PERT ((Abbasi and Mukattash, 2001; Foldes and Soumis, 1993; Mokhtari et al., 2010) or GERT (Arisawa and Elmaghraby, 1972); 2) cost function behaviour that can be discrete (Bregman, 2009), Linear continuous (Mitchell and Klastorin, 2007), non-linear convex (Berman, 1964) and linear-piecewise; 3) controllable variable in cost function which can be in the modes of activities with allocated budget (Abbasi and Mukattash, 2001) or resources (Sunde and Lichtenberg, 1995); 4) response variable in cost function which can be activity costs (Sunde and Lichtenberg, 1995) or its' durations (Godinho and Costa, 2007); 5) type of distribution function of cost and duration which can be Beta (Abbasi and Mukattash, 2001), Normal (Golenko-Ginzburg and Gonik, 1998) Exponential and Erlang (Azaron and

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Tavakkoli-Moghaddam, 2006); 6) objective function which can be maximizing project completion probability in a predefined deadline with limited budget (Abbasi and Mukattash, 2001), minimizing direct cost to obtain a pre-defined threshold of mean completion time (Foldes and Soumis, 1993), minimizing direct cost to reach a predefined threshold of project completion probability in a deadline (Mokhtari et al., 2010), minimizing the total cost, including direct and indirect approaches (Gutjahr et al., 2000), minimizing mean of project completion time and minimizing variance of completion time and cost (Azaron and Tavakkoli-Moghaddam, 2006) and minimizing mean of project completion time and minimizing mean of total project cost (Godinho and Costa, 2007); and 7) the solution approaches that contains exact approaches (Arisawa and Elmaghraby, 1972; Azaron and Tavakkoli-Moghaddam, 2006; Gutjahr et al., 2000), Heuristic approaches (Bregman, 2009; Foldes and Soumis, 1993) and meta-heuristic approaches (Aghaie and Mokhtari, 2009).

The above researches consider only two dimensions of the projects that are time and cost, but the other critical project dimension – quality – is completely missed. Babu and Suresh (1996) suggest that project quality may be affected by project crashing and develop linear programming models to study the time-cost-quality trade-off (TCQTP). Khang and Myint (1999) describe their attempt to apply the method to an actual cement factory construction project in Thailand. El-Rayes and Kandil (2005) design a model as a multi-objective genetic algorithm to transform the traditional TCTP to an advanced three-dimensional TCQTP. Iranmanesh et al. (2008) propose a discrete multi-mode model of TCQTP to deal with the problem. Pour et al. (2012) propose fuzzy logic theory to consider affecting uncertainty in project quality for discrete TCQTP. Zheng (2014) studies the fuzzy time-cost-quality trade-off problem for construction project and establishes a decision making model with multiple modes under resource-constrained environment. Golpîra and Hejazi (2014) propose a scenario based stochastic TCQTP over a multi-objective approach under this assumption that the probability of each scenario is available.

In this paper, we formulate a TCQTP based on scenario based stochastic optimization for multi-phase project scheduling problem that is, a well-known optimization attitude under uncertainty. SP applies probabilistic models to deal with uncertain data in terms of probability distributions. When accurate distributional information is available, stochastic programming has the advantage of incorporating this available distributional data; however, stochastic programming models are usually computationally more demanding (Hazır et al., 2010b). To do this, we use a goal programming approach in order to deal with the budget constraint. Our research and Golpîra and Hejazi (2014) address a TCQTP through the same methodology; however the approaches and the contributions of these papers are quite different. Golpîra and Hejazi (2014) introduce a stochastic approach and their model is not addresses the uncertainty of all the project costs unless the costs related directly to the duration of activities. But in this research, we use the approach that is introduced by Mulvey et al. (1995) in new point of view that uses a goal programming idea to obtain stochastic modelling. To the best our knowledge, this paper is the first research to introduce this approach. The remainder of this paper is organized as follows: Section 2 defines the proposed TCQTP. Section 3 presents an illustrative example to investigate the effectiveness of the developed method. Finally, the concluding remarks are reported in Section 4.

Problem statement

A scenario based stochastic multi-objective modelling for TCTP is as follows:

$$Minimiz\ z = \left(\sum_{s=1}^{S}\sum_{i=1}^{I} p_s\left(d_{si}^{+}+d_{si}^{-}\right)\right)+\left(\left(\sum_{p=1}^{P} p_s\sum_{s=1}^{S} a_{sp}x_{sp}\right)+\delta\left(\sum_{s=1}^{S} p_s\left(\Omega_s^{+}+\Omega_s^{-}\right)\right)\right) \quad (1)$$

$$s.t: \quad \sum_{p=1}^{P} x_{sp} - d_{si}^{+} + d_{si}^{-} = T_s \quad ,s=1,\dots,S,\ i=1,\dots,I \quad (2)$$

$$\sum_{p=1}^{P} c_{sp}x_{sp} - d_{si}^{+} + d_{si}^{-} = TC_s \quad ,s=1,\dots,S,\ i=1,\dots,I \quad (3)$$

$$\sum_{p=1}^{P} l_{sp}x_{sp} - d_{si}^{+} + d_{si}^{-} = Q_s \quad ,s=1,\dots,S,\ i=1,\dots,I \quad (4)$$

$$\sum_{p=1}^{P} a_{sp}x_{sp} - p_s\sum_{p=1}^{P}\sum_{s=1}^{S} a_{sp}x_{sp} = \Omega_s^{+} - \Omega_s^{-} \quad ,\text{s}=1,\dots,S \quad (5)$$

$$x_{sp} \geq 1, \quad s=1,\dots,S,\ i=1,\dots,I \quad (6)$$

$$d_{si}^{+}, d_{si}^{-} \geq 0, \quad s=1,\dots,S,\ i=1,\dots,I \quad (7)$$

$$\Omega_s^{+}, \Omega_s^{-} \geq 0, \quad s=1,\dots,S. \quad (8)$$

The index *s* is the scenario number, *i* is the constraint number and *p* is the project phase number. p_s is the probability of the scenarios and d_{si}^{+}, d_{si}^{-} are respectively the under-achievement and over-achievement of the i^{th} goal of the scenario set. x_{sp} is the duration of phase *p* over scenario *s*.

Constraint (1) represents the total cost of the project that is contains the expected amount of the deviational variables in the first term, and the human resource cost variability and model infeasibility penalty in the last term. Constraint (2) denotes the total estimated time of the project. T_s in this constraint, is the total time of the project over scenario *s*. Constraint (3) reveals the relation between total cost of the project and the duration of the phases of the project. The parameter TC_s in this constraint is the total cost of the project over scenario *s* except the costs of human resources. Constraint (4) explains the linkage between phase's duration and the quality that must be achieved. Q_s in this constraint is total quality achieved over scenario *s*. δ is the penalty that is assigned to control the deviation which may be accrued in constraint (5). This constraint contains the cost of human resources and the parameter a_{sp} is the per-unit human resource for phase p over scenario s of the project. Constraints (6)-(8) reveal the positivity of the variables. The two additional variables, Ω_s^+, Ω_s^- are interpreted as the amount by which the expected value of the human resource costs is less or more than the exact one according to the scenarios, respectively.

Simulation and results

To illustrate the usefulness and practicability of the proposed approach, an empirical case is studied. A real project containing three phases- planning, scheduling and controlling- is given as a case. The data for this study are collected in winter 2013 in Kurdistan that is previously addressed by Golpîra and Hejazi (2014). The data of the problem are illustrated in Table 1 and Table 2.

Table 1

Problem data of each scenario over each scenario

Phases	Probability of Scenarios	Scenario	Human and material resource monthly cost			Other monthly costs	Monthly cost of quality
1	0.40	1	29	26	27	20	30
		2	20	27	23	26	5
		3	15	27	15	30	20
2	0.35	1	19	10	13	20	24
		2	17	21	19	18	4
		3	10	21	21	25	16
3	0.25	1	15	13	22	18	22.5
		2	21	10	19	20	3.75
		3	17	16	14	40	15

Source: author's calculations based on Golpîra and Hejazi (2014)

Table 2

Problem data of the total project

Scenario	Total cost of the project	Timetable of the project	Total cost of quality of the project
1	1000	36	3.5
2	800	30	3
3	750	24	4

Source: author's calculations based on Golpîra and Hejazi (2014)

The problem is a linear stochastic goal programming which is simply solvable by Lingo software. The results are shown in Table 3. In this table symbol k is assigned to enumerate the number simulations.

Table 3

Solution data about the empirical exam

kth *simulation*	$\delta=0$			$\delta=1$		
	3	**2**	**1**	**3**	**2**	**1**
d_{11}^{+}	0	0	0	0	0	0
d_{11}^{-}	2.2	2.2	2.2	14.54	20.38	2.2
d_{12}^{+}	0	0	0	0	0	0
d_{12}^{-}	0	0	0	370.4	545.66	0
d_{13}^{+}	0	0	0	0	0	0
d_{13}^{-}	1.68	1.68	1.68	2.29	2.58	1.68
d_{21}^{+}	2.48	9.85	9.85	2.48	9.85	9.46
d_{21}^{-}	0	0	0	0	0	0
d_{22}^{+}	0	0	0	0	0	0
d_{22}^{-}	0	0	0	0	0	7.76
d_{23}^{+}	0	0	0	0	0	0
d_{23}^{-}	0.88	1.17	1.17	0.88	1.17	1.19
d_{31}^{+}	0	0	0	0	6.48	4.97
d_{31}^{-}	4.2	4.2	4.2	4.03	0	0
d_{32}^{+}	0	0	0	0	0	0
d_{32}^{-}	0	0	0	0	0	0
d_{33}^{+}	0	0	0	0	0	0
d_{33}^{-}	2.63	2.63	2.63	2.62	2.37	0
x_{11}	1	1	1	1	1	1
x_{12}	1	1	1	1	1	1
x_{13}	31.8	31.8	31.8	19.45	13.61	31.8
x_{21}	1	37.85	37.85	1	37.85	37. 46
x_{22}	1	1	1	1	1	1

k^{th} *simulation*	$\delta = 0$			$\delta = 1$		
	3	**2**	**1**	**3**	**2**	**1**
x_{23}	30.48	1	1	30.48	1	1
x_{31}	1	1	1	1.29	20.43	7.62
x_{32}	1	1	1	1	1	12.06
x_{33}	17.8	17.8	17.8	17.66	9.05	9.28
Q_1^+	111.67	322.83	57.43	0	0	0
Q_1^-	0	0	0	0	0	0
Q_2^+	0	0	62.48	0	0	0
Q_2^-	73.53	168.26	0	0	0	0
Q_3^+	0	0	0	0	0	0
Q_3^-	75.73	280.96	179.36	0	0	0
Objective	**418.7664**	**595.8841**	**476.6866**	**498.5399**	**654.0337**	**536.244**

Source: author's calculations based on Table 1 and Table 2

As one can see, the penalty coefficient δ which is assigned to control the human recourse cost variability is designed to be 0 or 1. If the δ is 0, the value of the variables Q_1 , Q_2 , Q_3 allowed to be nonzero as the human and material resource variability. But if the δ is 1, the value of the variables Q_1 , Q_2 , Q_3 are zero. In other words, it forces the variability of the human and material resource costs to be zero. On the other hand, if the manager wants to have a less duration, he/she can chose the scenario 1 that forces the project quality to be less than what expected, but makes the total costs to have no change in $\delta = 0$, and the human and material resource cost is forced to be more than its expected value. But it changes the total costs to be more than what expected, if the value of the δ is fixed on 1. So, the manager enforced to have more human and material resource costs to reach the better level of quality and project completion time. In scenario 2, if the value of the δ is 1, the manager encountered with extended project time and more cost of quality with no change in the human and resource cots and total costs. So, if the manager wants to have a fixed value of costs, he/she may encounter with some loss in quality and have extended project time. In scenario 3, all of the parameters are not worst in comparison with their expected values. In other words, if the manager selects the scenario 3, the trade-off is not needed and the project may be finished exactly.

Conclusion

In this paper, we consider the time-cost-quality trade-off problem in the field of project management under stochastic manner to handle the uncertainty of the real world project environment. The scenario approach is employed to deal with the problem. The results illustrate that the model exactly helps the decision maker to have some alternatives and in this decision making helps him/her to make better optimal trade-offs among all the critical factors of the project. In addition to handling uncertainty, the variety of scenarios that can be considered, numerous constraints that may be indicated in the model and its simplicity and solvability are making the model more flexible and practical in real worlds.

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